

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science; Bachelor of science in Applied Mathematics and Statistics

QUALIFICATION CODE: 07BSOC; 07BAMS

LEVEL: 5

COURSE NAME: CALCULUS 1

SESSION: NOVEMBER 2019

PAPER: THEORY

DURATION: 3 HOURS

MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER		
EXAMINER	Dr N. Chere and Mrs Y.Shaanika-Nkalle	
MODERATOR:	Prof Gunter Heimbeck	

INSTRUCTIONS		
1.	Answer ALL the questions in the booklet provided.	
2.	Show clearly all the steps used in the calculations.	
3.	All written work must be done in blue or black ink and sketches must	
	be done in pencil.	

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

SECTION A [Short answer questions] [$2\frac{1}{2}$ marks for each question]

QUESTION 1 [25 marks]

1. 1. Suppose that
$$\lim_{x\to 5} f(x) = 2$$
, $\lim_{x\to 5} g(x) = 6$. Then

1.1.1.
$$\lim_{x \to 5} (2f(x) - g(x)) = ----$$

1.1.2.
$$\lim_{x\to 5} \sqrt{(f(x))^4} = ----$$

1.1.3.
$$\lim_{x\to 5} \left(\frac{g(x)}{3f(x)}\right) = ----$$

1.1.4.
$$\lim_{x\to 5} (x^2 - 5(f(x))^2) = -----$$

1.2. Determine the following derivatives.

1.2.1.
$$\frac{d}{dx}(\ln(\sin x)) = -----$$

1.2.2.
$$\frac{d}{dx}(e^{\sqrt{x}}) = -----$$

1.2.3. If y = sin(sinx), then
$$\frac{dy}{dx}\Big|_{x=\frac{\pi}{2}} = -----$$

1.3. Suppose that f and g are continuous functions such that f(3) = 4 and

$$\lim_{x\to 3} (f(x) - 3g(x)) = 16$$
. Then the value of g (3) = -----

- 1.4. The domain of the function $f(x) = \sqrt{x^2 + x} = -$
- 1.5. Suppose a function f has the property that for all real numbers x, $3-|x| \le f(x) \le 3+|x|$. Then $\lim_{x\to 0} f(x) = -----$

SECTION B [Workout Problems]

QUESTION 2 [75]

2.1. Let
$$f(x) = -\sqrt{1 - 3x}$$
. Then

2.1.1. find a formula for
$$f^{-1}(x)$$
. [5]

2.1.2. state the domain of
$$f^{-1}$$
. [3]

2.2. Evaluate the following limits if it exists.

2.2.1.
$$\lim_{x \to -1^{-}} \left(\frac{1}{x+1} - \frac{1}{|x+1|} \right)$$
 [4]

2.2.2.
$$\lim_{x\to 0} \frac{2x + \tan 10x}{x}$$
 [6]

2.2.3.
$$\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$$
 [5]

2.2.4.
$$\lim_{x \to -3} \frac{x^3 + 27}{x^2 + 3x}$$
 [5]

2.3. Let
$$f(x) = \frac{1}{x+2}$$
. Find $f'(x)$ by using the limit definition of derivative. [5]

2.4. Use the precise definition of limit to prove that
$$\lim_{x\to 2} (4-6x) = -8$$
. [7]

2.5. Use the chain rule to find
$$\frac{dy}{dx}$$
 if $y = \left(\frac{x^2}{x+1}\right)^3$ [5]

- 2.6. If $x+y+xy^2=3$, then
- 2.6.1. Use implicit differentiation to solve and express $\frac{dy}{dx}$ in terms of x and y. [5]
- 2.6.2. use the result in (2.6.1) to find an equation of a tangent line to the curve $x+y+xy^2=3$ at (1,1).
 - 2.7. Suppose $f(x) = 2x^5 + x^3 + 1$. Then

2.7.1. find
$$(f^{-1})'(x)$$
 [5]

2.7.2. use (2.7.1) to find
$$(f^{-1})'(4)$$
 [2]

- 2.8. Let $f(x) = 3x^4 4x^3$. Then find
- 2.8.1. the local maximum and local minimum value of f if there are any. [5]
- 2.8.2. the intervals on which f is increasing and on which f is decreasing. [4]
- 2.8.3. the open intervals on which f is concave upward and on which it is concave downward.

[4]

2.8.4. the inflection point(s). [2]

END OF EXAMINATION